

The decay-time of non-commutative micro-black holes

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ABSTRACT: Non-commutative black holes are characterised by a minimum mass which would result in a remnant after the Hawking evaporation ends. We numerically study the decay of neutral non-commutative black holes for up to ten spatial dimensions and typical parameters that would make their production possible at the LHC. Neglecting possible accretion mechanism, we find that decay-times are extremely short.

KEYWORDS: Large Extra Dimensions, Black Holes, Models of Quantum Gravity.

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1. Introduction

One of the most striking predictions of General Relativity is the existence of black holes, that is event horizons which surround and hide the point-like or otherwise singular classical sources. On general quantum-mechanical arguments, one however expects that singular sources are smeared out and there are indeed many theoretical reasons to believe the fabric of space-time can be effectively described by non-commutative geometry at short scales (see, for example section 2 of ref. [1] and ref. [2]). Such modifications to General Relativity must necessarily have an impact on the structure of black holes.

The non-commutative analogue of the Schwarzschild black hole metric was found in ref. [3] (see also ref. [4]) and subsequently generalised to include the electric charge [5] and extra-spatial dimensions [6, 7] and the case of a traversable wormhole [8]. A common feature for all such metrics is that there exists a minimum mass for the black hole which solely depends on the non-commutative length ℓ^1 and the fundamental scale of gravity $M_g = \ell_g^{-1}$ (for a comprehensive and updated review, see ref. [1]). The former is usually identified with the Planck length ℓ_p and the latter with the Planck mass M_p in four space-time dimensions, but ℓ could actually be much larger and M_g much lower, possibly around 1 TeV, if extra-dimensions exists [9, 10]. This has opened up the possibility of having micro-black holes (mBH) with a mass of a few TeV's [11] that might be produced and detected at the Large Hadron Collider (LHC) [12–14].

The standard picture of mBH at the LHC is that they should be produced with a mass of around 10 TeV and very rapidly decay completely [12] via the Hawking effect [15]. Generally small (but possibly significant) corrections to this scenario arise from the use of the microcanonical picture [16], which essentially enforces energy conservation throughout the evaporation process and suppresses the emission for small mBH mass [13, 17]. If the non-commutative description of mBH given in ref. [1] is correct, one however expects that

¹This length is related to the non-commutative parameter θ of ref. [1] by $\ell = 2\sqrt{\theta}$.

a (stable) remnant will be left at the end of the Hawking evaporation [18], which might exit the accelerator and detectors. In this context, and for phenomenological purposes, it is therefore important to estimate the typical decay-times for such objects. For example, the issue of mBH life-times has recently been debated in refs. [19].

Since it is widely accepted that mBH discharge very quickly, via several processes, including the Schwinger mechanism at least for $3 \leq d \leq 5$ [7], we shall here consider only the neutral metrics of refs. [1, 6] for $d = 5, \dots, 10$ spatial dimensions (noting that higher values of d seem to be favoured [20]). In order to have mBH produced at the LHC, we need a minimum mBH mass of order 1 TeV, which in turn implies that $\ell_g \sim \ell$. Further, we shall see that the typical temperature of such mBH's always remains smaller than their mass, and microcanonical corrections can thus be neglected. The evolution of the mBH mass is then determined by solving the relevant equation numerically for initial mBH mass of order 10 TeV and an upper bound for the decay-time obtained.

We shall use units with $c = \hbar = k = 1$.

2. Non-commutative neutral mBH

We recall here that the non-commutative metric of a neutral black hole of (asymptotic) proper mass m in $d + 1$ space-time dimensions is given by [7]

$$ds^2 = -A_d dt^2 + A_d^{-1} dr^2 + r^{d-1} d\Omega_{d-1}^2, \tag{2.1}$$

where

$$A_d = 1 - \frac{2G_d m}{r^{d-2} \Gamma(d/2)} \gamma\left(\frac{d}{2}; \frac{r^2}{\ell^2}\right), \tag{2.2}$$

and

$$\gamma(a; x) = \int_0^x u^{a-1} e^{-u} du. \tag{2.3}$$

With $d = 3$ spatial dimensions, the Newton constant $G_3 = G_N = \ell_p/M_p = \ell_p^2$ and one usually assumes that $\ell \simeq \ell_p$. In the presence of extra spatial dimensions, $d > 3$, we can likewise write $G_d = \ell_g^{d-2}/M_g = \ell_g^{d-1}$, but keeping ℓ and ℓ_g distinct. Of course, the non-commutative length must be short enough to agree with present experimental bounds on the validity of the Newton law at short distance, that is $\ell \lesssim 1 \mu\text{m}$ [21] and for the gravitational mass we shall assume $M_g \simeq 1 \text{ TeV}$.

The radial mass-energy function for the above solution is given by

$$m(r) = \frac{m}{\Gamma(d/2)} \gamma\left(\frac{d}{2}; \frac{r^2}{\ell^2}\right), \tag{2.4}$$

and

$$\lim_{r \rightarrow \infty} m(r) = m, \tag{2.5}$$

as anticipated. Further, for sufficiently large proper mass, there exists two horizons, the outer one being located at $r = r_+$, which is related to m by

$$M_d = \ell_g^{d-2} \frac{m}{M_g} = r_+^{d-2} \frac{\Gamma(d/2)}{2 \gamma\left(\frac{d}{2}; \frac{r_+^2}{\ell^2}\right)}, \quad (2.6)$$

where $\Gamma(a)$ denotes the Gamma function and $M_d = G_d m$ the Arnowitt-Deser-Misner (ADM) mass of the black hole ². The inverse Hawking temperature (in geometrical units) is finally given by

$$\beta_d = 4 \pi r_+ \left[(d-2) - 2 \left(\frac{r_+}{\ell}\right)^d \frac{e^{-r_+^2/\ell^2}}{\gamma\left(\frac{d}{2}; \frac{r_+^2}{\ell^2}\right)} \right]^{-1}. \quad (2.7)$$

3. Hawking evaporation

In order to study the evaporation of a mBH, we need to estimate the corresponding Hawking flux Φ , that is the amount of energy emitted through a unit area per unit time in the form of thermal radiation. The latter is described by quantum fields propagating on the $(d+1)$ -dimensional background (2.1) and we recall that the space-time non-commutativity makes the field theory UV finite, thanks to the presence of a damping term in the momentum-space propagator, namely $G(p) \sim \exp(-\ell^2 p^2/8)$ (for the details, see ref. [22]). As a result, considering for simplicity the case of a massless scalar field and neglecting the grey-body factor, one finds

$$\Phi = 2 \int \frac{d^d p}{(2\pi)^d} \frac{e^{-\frac{1}{8} \ell^2 p^2} p}{e^{\beta_d p} - 1}. \quad (3.1)$$

From this expression, one can then obtain the total luminosity \mathcal{L}_d which governs the time dependence of the mBH proper mass in the canonical picture [7],

$$-\frac{dm}{dt} \propto F_d(r_+) \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left(\frac{\ell}{2\beta_d}\right)^{2n} \Gamma(2n+4) \zeta(2n+4) \equiv \mathcal{L}_d(r_+), \quad (3.2)$$

where $\zeta(a)$ denotes the Riemann zeta function.

The exact form of the function F_d in the above expression is actually difficult to determine and for a number of reasons. To begin with, one should consider the contribution of all the different kinds of particles that can be emitted, like Standard Model fermions and bosons as well as (bulk and brane) gravitons. Further, particles with different spins propagate differently and their emission is thus suppressed by spin-dependent grey-body factors, which, in turn, strongly depend on the precise near-horizon geometry. In this respect, there are arguments which suggest that Standard Model particles propagating only in our $(3+1)$ -dimensional space-time makes for most of the Hawking radiation [23], so that $F_d \propto r_+^2$ (the area of the brane-section of the horizon), as well as arguments against

²Note that M_d has dimensions of $(\text{length})^{d-2}$.

this scenario [24], according to which one should instead have $F_d \propto r_+^{d-1}$ (the area of the bulk horizon, neglecting a possible squeezing [25]). Finally, one realistically expects that mBH at the LHC would be produced with an intrinsic spin (see, for example, ref. [26]), but no rotating analogue of the metric (2.1) is known to date.

Since our main goal here is to provide order of magnitude estimates of the typical decay-times, we shall consider three possibilities:

B) The mBH emits all the particles in the entire bulk:

$$F_d^{\text{B}} = r_+^{d-1} \beta_d^{-(d+1)} . \tag{3.3}$$

b) The mBH emits like a four-dimensional black hole except for the modified (inverse) temperature:

$$F_d^{\text{b}} = r_+^2 \beta_d^{-4} . \tag{3.4}$$

bB) The mBH evaporates with a dependence on the temperature like that in eq. (3.3) and on the 2-dimensional horizon area like in eq. (3.4):

$$F_d^{\text{bB}} = \ell_g^{d-3} r_+^2 \beta_d^{-(d+1)} . \tag{3.5}$$

All the above forms will be used for the numerical simulations reported on later.

In order to solve (3.2) numerically, we introduce dimensionless quantities (denoted by a bar) by expressing everything in units of ℓ (to a suitable power). For example, the dimensionless ADM mass will be given by

$$\bar{M}_d = M_d \ell^{2-d} , \tag{3.6}$$

and the proper mass by

$$\bar{m} = m \ell = \bar{M}_d \left(\frac{\ell}{\ell_g} \right)^{d-1} . \tag{3.7}$$

We shall sometimes find it more convenient to express the proper mass m in units of M_g (denoted by a tilde) as

$$\tilde{m} = \frac{m}{M_g} = \bar{M}_d \left(\frac{\ell}{\ell_g} \right)^{d-2} . \tag{3.8}$$

Since we assume $M_g \simeq 1$ TeV, \tilde{m} will just be the mBH mass in TeV's. Next, we shall need to truncate the series in eq. (3.2) to a maximum integer value $n = n_{\text{max}}$. With the above assumptions and redefinitions, eq. (3.2) becomes

$$\frac{d\bar{m}}{d\bar{t}} = -C F_d(\bar{r}_+) \sum_{n=0}^{n_{\text{max}}} \frac{(-1)^n}{4^n n!} \bar{\beta}_d^{-2n} \Gamma(2n+4) \zeta(2n+4) \equiv -\bar{\mathcal{L}}_d(\bar{r}_+) , \tag{3.9}$$

where C is a numerical constant which we shall comment upon later. Finally, by making use of eq. (2.6), we can obtain an equation which contains the horizon radius as the only time-dependent variable,

$$\frac{d\bar{r}_+}{d\bar{t}} = -\frac{d\bar{r}_+}{d\bar{m}} \bar{\mathcal{L}}_d(\bar{r}_+) = -\frac{\bar{\beta}_d \gamma\left(\frac{d}{2}; \bar{r}_+^2\right)}{2\pi \bar{r}_+^{d-2} \Gamma(d/2)} \left(\frac{\ell_g}{\ell}\right)^{d-1} \bar{\mathcal{L}}_d(\bar{r}_+), \quad (3.10)$$

which we now proceed to study numerically, with the aim of determining the evolution of a mBH with initial mass $m(t=0) \simeq 10$ TeV.

4. Numerical results

Upon employing the three forms listed in the previous section for the function F_d , we always found the same qualitative picture. In particular, for fixed C , the choice of F_d^B in eq. (3.3) always produced the longest decay-times for $5 \leq d \leq 10$. Moreover, for a given initial mBH mass and choice of F_d , the decay-time increases for increasing d and is (roughly) linearly proportional to C . Since larger values of d are anyway favoured [20], we shall therefore show the detailed analysis just for the case $d = 10$ and $F_{10} = F_{10}^B$ with $C = 1$. This choice yields the longest decay-time overall (for fixed $C = 1$), which can thus serve as a “worst” (or “best”) case scenario, and we shall further comment about other cases when relevant.

First of all, we need the minimum mBH mass. This is obtained by first minimising the ADM mass (2.6) with respect to r_+ , which, for $d = 10$, yields the minimum (dimensionless) horizon radius $\bar{r}_{\min} \simeq 1.11$, corresponding to a minimum ADM mass $\bar{M}_{\min} \simeq 133$. In order for the latter to translate into a minimum proper mass $m_{\min} \simeq 1.2$ TeV, we thus set (see eq. (3.8))

$$\ell_g \simeq 1.8 \ell. \quad (4.1)$$

For smaller values of d , one obtains slightly smaller ℓ_g 's, but with a very weak dependence on the space dimension. In fact, for $d = 5$, one has the smallest value $\ell_g \simeq 1.4 \ell$.

In figure 1, we then plot the mBH temperature $\bar{\beta}_{10}^{-1}$ (left panel) and its ratio with the proper mass \bar{m} as functions of the horizon radius \bar{r}_+ (right panel). Since the latter ratio becomes smaller for lower d , it is clear that the mBH mass always remains significantly larger than the typical energy of emitted quanta (proportional to the temperature) and the canonical expression (3.2) applies throughout for all values of d (and all choices of F_d).

Next, we need to determine how many terms in the sum in eq. (3.10) we should keep. In figure 2, left panel, we show the dimensionless luminosity in $d = 10$ with $F_{10} = F_{10}^B$ and $C = 1$ for $n_{\max} = 0, 2$ and 10 around the peak, where discrepancies are the largest. We can thus conclude that it is well sufficient to use $n_{\max} = 2$ (the relative difference with respect to $n_{\max} = 10$ does not depend on the choice of F_d and C and is less than $8 \cdot 10^{-3}$; see right panel). For smaller values of d , this approximation actually becomes even better. For example, for $d = 5$ one obtains the smallest relative error of less than $6 \cdot 10^{-5}$.

From eq. (2.6) and (3.8), we find that an initial mBH proper mass of about 10 TeV corresponds to an horizon radius $\bar{r}_+ \simeq 2.5$ for $d = 10$. With this initial condition, eq. (3.10)

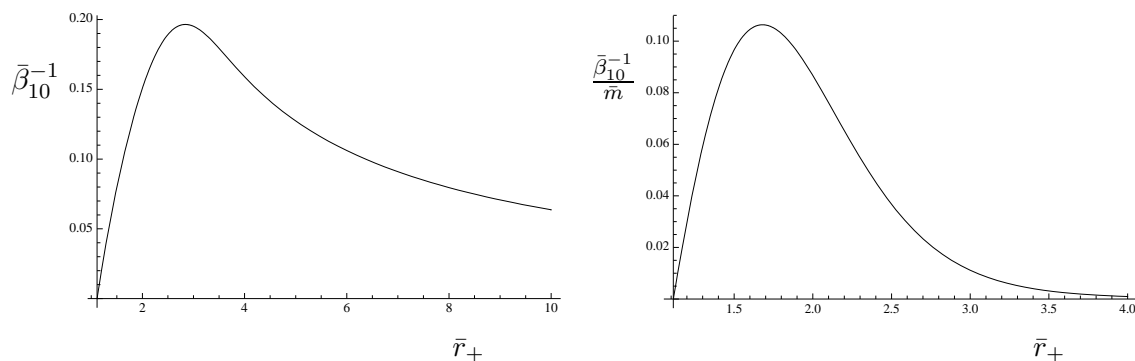


Figure 1: Temperature (left panel) and its ratio with proper mass (right panel) vs horizon radius in $d = 10$ (where $\bar{r}_+ \geq 1.11$). The maximum temperature $\bar{\beta}_{10}^{-1} \simeq 0.11 \bar{m}$ is reached for $\bar{r}_+ \simeq 1.68$.

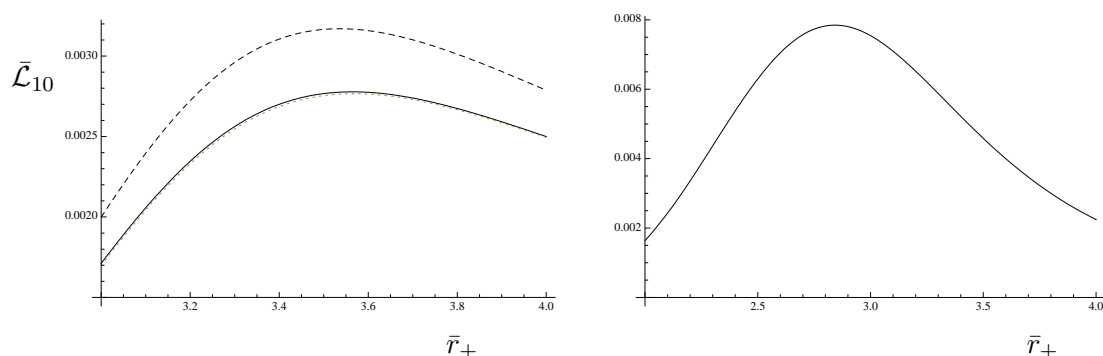


Figure 2: Left panel: approximate mBH luminosity for $n_{\max} = 0$ (dashed line), 2 (solid line) and 10 (dotted line). Right panel: relative difference in luminosity for $n_{\max} = 2$ and $n_{\max} = 10$.

with $C = 1$ and $F_{10} = F_{10}^B$ yields the solution plotted in the left panel of figure 3, from which the time-dependent proper mass can be easily obtained (right panel). The latter plot shows that the decay occurs through several stages: it is initially rather slow (for $0 \leq \bar{t} \lesssim 300$), subsequently becomes much faster (for $300 \lesssim \bar{t} \lesssim 10^6$) and finally approaches the remnant again slowly (for $\bar{t} \gtrsim 10^6$). For our choice of $\ell_g = 1 \text{ TeV}^{-1} \simeq 1.8 \ell$, we obtain that $\ell \simeq 0.6 \text{ TeV}^{-1} \simeq 3 \cdot 10^{-28} \text{ sec}$ and the estimated decay-time $t_{10} \simeq 10^{10} \ell \simeq 10^{-18} \text{ sec}$ is practically instantaneous. This result changes rather weakly with d , decreasing down to the minimum $t_5 \simeq 10^6 \ell \simeq 10^{-22} \text{ sec}$. As we mentioned before, different choices of F_d produce slightly different decay curves, but do not yield longer decay-times. For example, for $d = 5$ and $F_5 = F_5^b$ in eq. (3.4) one obtains the shortest possible value for $C = 1$, that is $t_5 \simeq 10^4 \ell \simeq 10^{-24} \text{ sec}$. For different values of C , all the decay-times scale roughly linearly. Assuming $C \gtrsim 10^{-2}$ and considering the case $d = 10$ with $F_{10} = F_{10}^B$ thus yields the fairly conservative upper bound

$$t_{\text{decay}} \lesssim 10^{-16} \text{ sec}, \tag{4.2}$$

which is extremely short compared to the sensitivity of present detectors (on the order of hundreds of picoseconds).

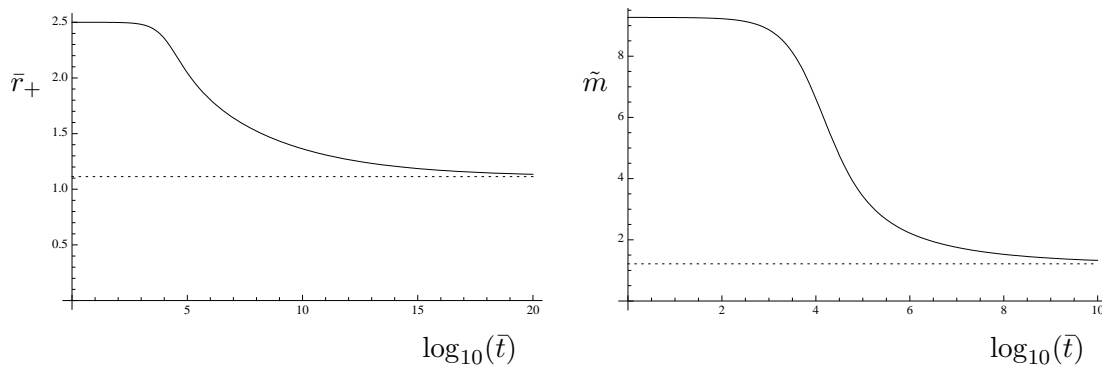


Figure 3: Left panel: horizon radius for initial mBH mass $m \simeq 9.2$ TeV (solid line) and its minimum value (dotted line). Right panel: mBH proper mass for same initial condition (solid line) and remnant mass $m_{\text{min}} \simeq 1.2$ TeV (dotted line).

5. Conclusions

We have estimated the decay-times of non-commutative mBH’s that might be produced at the LHC and found that they should evaporate nearly instantaneously (see the upper bound in eq. (4.2)), much the same as is expected according to the “canonical” scenario of ref. [12]. We can consequently conclude that non-commutative mBHs would evaporate within the detectors, since they can propagate at most a few nanometers away from their point of production during the evaporation.

The present results are similar to previous estimates of decay-times for usual (“commutative”) mBH’s in the ADD brane-world [9], described according to the microcanonical picture [16], for which life-times were obtained on the order of 10^{-17} sec or shorter [13]. Remarkably, our conclusions are also comparable with the recently estimated life-times of mBH’s derived from Generalised Uncertainty Principles in ref. [27]. Actually, this is not surprising, since the dependence of the temperature on the mBH mass is roughly similar in all of these cases [17]. Of course, the mBH’s of ref. [13] evaporate completely (thus reaching zero temperature for vanishing mass) and their life-time is actually dominated by the latest stages (when $m \lesssim 5$ TeV), whereas the end-point of the process here, as well as in ref. [27], is a (presumably) stable remnant of mass $m_{\text{min}} \simeq 1$ TeV and zero Hawking temperature.

Let us conclude by mentioning that no accretion mechanism has been included in our analysis of the evaporation, which is fully consistent given the very short decay-times. What happens afterwards and the final fate of the remnant are still open questions.

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